GRAVITATION

(1) $V_c = \sqrt{\frac{GM}{R}}$

$= \sqrt{\frac{G V \rho}{R}} = \sqrt{\frac{G \cdot \frac{4}{3}\pi R^3 \cdot \rho}{R}}$

$= \sqrt{\frac{4}{3} G \pi R^2 \rho} = 2R\sqrt{\frac{\pi \rho G}{3}}$

(2) Data :

$T_1 = 365$ days

$T_2 = ?$

$r_1 = x$

$r_2 = 2x$

Solution :

$T^2 \propto r^3$

$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$

$\left(\frac{365}{T_2}\right)^2 = \left(\frac{\cancel{x}}{2\cancel{x}}\right)^3$

$T_2 = 1032$ days

(3) Data :

$T = 24$ hrs.

$h = ?$

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$M = 6 \times 10^{24}$ kg

$R = 6400 \times 10^3$ m

Solution :

$T^2 = \frac{4\pi^2 r^3}{GM}$

$(24 \times 3600)^2 = \frac{4(3.142)^2 (6400 \times 10^3 + h)^3}{6.67 \times 10^{-11} \times 6 \times 10^{24}}$

$h = 35.89 \times 10^6$ m

(4) $T^2 \propto r^3$

$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$

$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{10^{13}}{10^{12}}\right)^3$

$\frac{T_1}{T_2} = \sqrt{1000}$

$\frac{T_1}{T_2} = 31.62$

Ans : 31.62 : 1

$\frac{V_{C1}}{V_{C2}} = \sqrt{\frac{r_2}{r_1}}$

$\frac{V_{C1}}{V_{C2}} = \sqrt{\frac{10^{12}}{10^{13}}}$

$\frac{V_{C1}}{V_{C2}} = \sqrt{\frac{1}{10}}$

$\frac{V_{C1}}{V_{C2}} = 0.3162$

Ans: 0.3162 : 1

(5) Date

$M_e = 3.5$ kg wt

$= 3.5 \times 9.8$ N

$M_p = \frac{M}{7}$

$R_p = \frac{R}{2}$

Solution:

$$\frac{g}{g_p} = \frac{\cancel{G}M}{R^2} \times \frac{R_p^2}{\cancel{G}M_p}$$

$$\frac{g}{g_p} = \frac{M}{R^2} \times \frac{(R/2)^2}{(M/7)}$$

$$\frac{g}{g_p} = \frac{M}{R^2} \times \frac{R^2}{M} \times \frac{7}{4}$$

$$\frac{g}{g_p} = \frac{7}{4}$$

$$g_p = \frac{4}{7} g$$

Wt. on planet $= 3.5 \times \frac{4g}{7}$

$= \frac{3.5 \times 4 \times 9.8}{7}$ N

Equivalent on earth

$= \frac{3.5 \times 4 \times 9.8}{7} \times \frac{1}{9.8}$ kgwt

$= 2$ kgwt.

(6) Date

$\frac{r_1}{r_2} = \frac{3}{8}$

$\frac{V_{c1}}{V_{c2}} = ?$

$\frac{T_1}{T_2} = ?$

Solution:

$$\frac{V_{c1}}{V_{c2}} = \sqrt{\frac{GM}{r_1}} \times \sqrt{\frac{r_2}{GM}}$$

$$\frac{V_{c1}}{V_{c2}} = \sqrt{\frac{r_2}{r_1}} = \sqrt{\frac{8}{3}}$$

$= 1.633 : 1$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^3$$

$$\frac{T_1}{T_2} = \left(\frac{3}{8}\right)^{3/2}$$

$T_1 : T_2 = 0.2296 : 1$

(7) Data

$V_e = ?$

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$R = 6400 \times 10^3\ m$

$g = 9.8\ m/s^2$

Solution

$$V_e = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2gR^2}{R}}$$

$$\therefore V_e = \sqrt{2gR}$$

$$V_e = \sqrt{2 \times 9.8 \times 6400 \times 10^3}$$

$$V_e = 11200\ m/s$$

$$\therefore V_e = 11.2\ km/s$$

(8) Data

$BE = ?$

$m = 50\ kg$

$h = 0\ m$

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$R = 6400 \times 10^3\ m$

$M = 6 \times 10^{24}\ kg$

Solution

$$BE = \frac{GMm}{R}$$

$$= \frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 5}{6400 \times 10^3}$$

$$= 3.127 \times 10^9\ J$$

(9) Data

$TE = ?$; $BE = ?$

$m = 1000\ kg$

$h = 1600 \times 10^3\ m$

$G = 6.67 \times 10^{-11}\ Nm^2/kg^2$

$R = 6400 \times 10^3\ m$

$M = 6 \times 10^{24}\ kg$

Solution

$$TE = -\frac{GMm}{2(R+h)}$$

$$= -\frac{6.67 \times 10^{-11} \times 6 \times 10^{24} \times 1000}{2(6400 \times 10^3 + 1600 \times 10^3)}$$

$$= -2.501 \times 10^{10}\ J$$

$$BE = -TE$$

$$= 2.501 \times 10^{10}\ J$$

(10) $r_1 = x$

$r_2 = x/2$

$$T^2 \propto r^3$$

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

$$\frac{1^2}{T_2^2} = \frac{x^3}{(x/2)^3}$$

$$T_2 = \sqrt{\frac{1}{8}} = 0.3536\ yr$$

$$\cong 129\ days$$

11) $\longleftarrow 6.4\times10^{6}\,m \longrightarrow$

5 kg $\qquad$ 6×10^{24} kg

$$f = \frac{G m_1 m_2}{r^2}$$

$$= \frac{6.67\times10^{-11}\times5\times6\times10^{24}}{(6.4\times10^{6})^2}$$

$$= 48.85\ N$$

$$F_1 = m_1 a_1$$

$$\therefore a_1 = \frac{98.85}{5} = 9.77\ m/s^2$$

$$f = m_2 a_2$$

$$a_2 = \frac{48.85}{6\times10^{24}}$$

$$= 8.142\times10^{-24}\ m/s^2$$

12) ~~g~~ Data

$M = 6\times10^{24}$ kg

$R = 6400\times10^{3}$ m

$g = 9.774\ m/s^2$

Solution:

$$g = \frac{GM}{R^2}$$

$$\therefore G = \frac{gR^2}{M} = \frac{9.774\times(6.4\times10^{6})^2}{6\times10^{24}}$$

$$= 6.672\times10^{-11}\ Nm^2/kg^2$$

13) Data: $g = 9.8\ m/s^2$

$G = 6.673\times10^{-11}\ Nm^2/kg^2$

$R = 6.4\times10^{6}$ m

Solution:

$$\rho = \frac{M}{V}$$

$$\rho = \frac{gR^2/G}{\frac{4}{3}\pi R^3}$$

$$= \frac{3}{4}\,\frac{g}{\pi G R}$$

$$= \frac{3}{4}\times\frac{9.8}{3.142\times6.673\times10^{-11}\times6\times10^{24}}$$

$$= 5477\ kg/m^3$$

14) Data $m = 100$ kg

$h = 1000\times10^{3}$ m

(i) $m_h = 100$ kg.

(ii) $$\frac{g_h}{g} = \frac{R^2}{(R+h)^2}$$

$$\therefore g_h = \frac{9.8\times(6.4\times10^{6})^2}{(6.4\times10^{6}+10^{6})^2}$$

$$= 7.330\ m/s^2$$

$$\therefore wt = m g_h$$

$$= 100\times7.330$$

$$= 733\ kg$$